

## Chapter Summary

### Chapter Summary: Motion in Two and Three Dimensions

#### Displacement

When a particle changes position, it has a displacement  $\vec{d}$  given by the following equation.

$$\vec{d} = \Delta\vec{r} = \vec{r}(t + \Delta t) - \vec{r}(t) \quad \boxed{4.1}$$

where its final position occurs at time  $t + \Delta t$  and its initial position is at time  $t$ . We generally use the coordinates  $x$ ,  $y$ , and  $z$  to locate a particle at point  $\mathbf{P}(x, y, z)$  in three dimensions. If the particle is moving, the variables  $x$ ,  $y$ , and  $z$  are functions of time:

$$x = x(t) \quad y = y(t) \quad z = z(t) \quad \boxed{4.2}$$

The **position vector** from the origin of the coordinate system to point  $\mathbf{P}$  is  $\vec{r}(t)$ . In unit vector notation, introduced in [Coordinate Systems and Vector Components](#),  $\vec{r}(t)$  is

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \quad \boxed{4.3}$$

#### Velocity

The instantaneous velocity represents the rate of change of a particle's position in the limit of the time interval becoming infinitesimally small.

$$\vec{v}(t) = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt} \quad \boxed{4.4}$$

The *S/* units for instantaneous velocity are meters/second (m/s). [Equation 4.4](#) can also be written in terms of the components of  $\vec{v}(t)$ . Since  $\vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} + z(t) \hat{k}$ , we can write

We can also write the average velocity for a particle, such as the one in the above figure as follows.

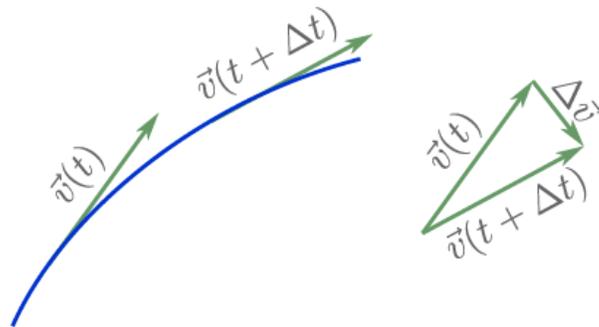
The average velocity of a particle is equal to its final position at a time  $t_2$  minus its initial position at time  $t_1$  divided by the elapsed time.

$$\begin{aligned} \vec{v}_{\text{ave}} &= \frac{\Delta \vec{r}}{\Delta t} \\ &= \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1} \end{aligned} \quad \boxed{4.6}$$

The *S/* units for average velocity are meters/second (m/s).

In two and three dimensions, in addition to obtaining the displacement and velocity vectors of an object in motion, we often want to know its **acceleration** vector at any point in time along its trajectory. This is useful for describing motion when the velocity is not constant, such as when an object is speeding up or slowing down or just changing direction.

The figure below shows two points along the trajectory of motion for a particle. The velocity of the particle is changing with elapsed time, which is an acceleration.



$$t_{\text{tof}} = \frac{2v_{y,i}\sin\theta}{g}$$

This equation does not apply when the projectile lands at a different elevation than it was launched, as we saw in the example above of the tennis player hitting the ball into the stands. The other solution,  $t = 0$  s, corresponds to the time at launch. The time of flight is linearly proportional to the initial velocity in the  $y$ -direction and inversely proportional to the acceleration due to gravity. Thus, on the Moon, where gravity is approximately one-sixth that of Earth, a projectile launched with the same velocity as on Earth would be airborne six times as long.

## Trajectory

The trajectory of a projectile can be found by eliminating the elapsed time variable from the kinematic equations and solving for  $y(x)$ . We take  $x_i = y_i = 0$  so the projectile is launched from the origin. The kinematic equation for the horizontal displacement is,

$$x = v_{x,i}t \\ \Rightarrow t = \frac{x}{v_{x,i}} = \frac{x}{v_i\cos\theta}$$

Substituting the expression for the elapsed time into the equation for the position  $y = (v_i\sin\theta)t - \frac{1}{2}gt^2$  gives

The **time of flight** for a projectile that is launched from and lands on a flat, horizontal surface at the same height is

$$t \left( v_{y,i}\sin\theta - \frac{gt}{2} \right) = 0$$

Multidimensional motion with constant acceleration can be treated the same way as shown in the previous chapter for one-dimensional motion. Earlier, we showed that three-dimensional motion is equivalent to three one-dimensional motions, each along an axis perpendicular to the others. To develop the relevant equations in each direction, let's consider the two-dimensional problem of a particle moving in the  $xy$ -plane with constant acceleration, ignoring the  $z$ -component for the moment. The acceleration vector is  $\vec{a} = a_x \hat{i} + a_y \hat{j}$ . For situations involving a constant acceleration in one or more dimensions, the equations of kinematics can be applied. Here we rewrite the equations using appropriate subscripts. Remember, the elapsed time  $t$  is the same for both dimensions.

The When the acceleration is constant, then there is no distinction between the average and the instantaneous acceleration.

### Constant Acceleration

$$\vec{a}(t) = \frac{d^2x(t)}{dt^2} \hat{i} + \frac{d^2y(t)}{dt^2} \hat{j} + \frac{d^2z(t)}{dt^2} \hat{k} \quad \boxed{4.10}$$

Also, since the velocity is the derivative of the position function, we can write the acceleration in terms of the second derivative of the position function.

$$\vec{a}(t) = \frac{dv_x(t)}{dt} \hat{i} + \frac{dv_y(t)}{dt} \hat{j} + \frac{dv_z(t)}{dt} \hat{k} \quad \boxed{4.9}$$

The acceleration in terms of components is

$$\begin{aligned} \vec{a}(t) &= \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt} \end{aligned} \quad \boxed{4.8}$$

The SI units for acceleration are  $\mathbf{m/s^2}$ .

The instantaneous acceleration is obtained in the limit where the change in time approaches zero, and the two points along the trajectory move closer together.

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} \quad \boxed{4.7}$$

The change in the velocity is equal to the final velocity minus the initial velocity.

During a time interval  $\Delta t = t_f - t_i$ , the **average acceleration** is determined using the final and initial velocity vectors.

$$\vec{v}(t) + \Delta\vec{v} = \vec{v}(t + \Delta t)$$

Then,  $\Delta\vec{v} = \vec{v}(t + \Delta t) - \vec{v}(t)$

From the right side of the figure above, we see that the final velocity is found by adding the change in the velocity to the initial velocity.

## INDEPENDENCE OF MOTION

In the kinematic description of motion in two dimensions, we are able to treat the horizontal and vertical components of motion separately. In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa. This principle also extends to the third dimension.

### Acceleration

where  $v_x(t) = \frac{dx(t)}{dt}$ ,  $v_y(t) = \frac{dy(t)}{dt}$ ,  $v_z(t) = \frac{dz(t)}{dt}$ . Note that the Cartesian unit vectors  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are constants and, as such, are time-independent.

$$\vec{v}(t) = v_x(t) \hat{i} + v_y(t) \hat{j} + v_z(t) \hat{k} \quad \boxed{4.5}$$

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